Variable Names and Descriptions (continued)

K	Effective length factor of column
L	Length of column or beam
M	Applied moment
Mx	Internal bending moment at x
P	Load (Eccentric Columns), or Point load (beams)
Pcr	Critical load
r	Radius of gyration
V	Shear force at x
w	Distributed load
x	Distance along beam
y	Deflection at x

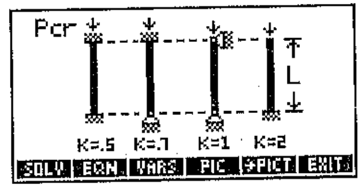
For simply supported beams and cantilever beams ("Simple Deflection" through "Cantilever Shear"), the calculations differ depending upon the location of x relative to the loads.

- Applied loads are positive downward.
- The applied moment is positive counterclockwise.
- Deflection is positive upward.
- Slope is positive counterclockwise.
- Internal bending moment is positive counterclockwise on the left-hand part.
- Shear force is positive downward on the left-hand part.

Reference: 2.

Elastic Buckling (1, 1)

These equations apply to a slender column ($K \cdot L/r > 100$) with length factor K.



Equations:

$$\mathrm{Pcr} = \frac{\pi^2 \cdot \mathrm{E} \cdot \mathrm{A}}{\left(\frac{\mathrm{K} \cdot \mathrm{L}}{\mathrm{r}}\right)^2} \qquad \mathrm{Pcr} = \frac{\pi^2 \cdot \mathrm{E} \cdot \mathrm{I}}{\left(\mathrm{K} \cdot \mathrm{L}\right)^2} \qquad \sigma\mathrm{cr} = \frac{\mathrm{Pcr}}{\mathrm{A}} \qquad \mathrm{r} = \sqrt{\frac{\mathrm{I}}{\mathrm{A}}}$$

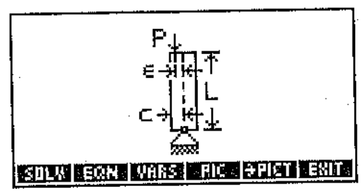
Example:

Given: L=7.3152_m, r=4.1148_cm, E=199947961.502_kPa, A=53.0967_cm^2, K=0.7, I=8990598.7930_mm^4.

Solution: Pcr=676.6019_kN, σcr=127428.2444_kPa.

Eccentric Columns (1, 2)

See "Elastic Buckling."



Equations:

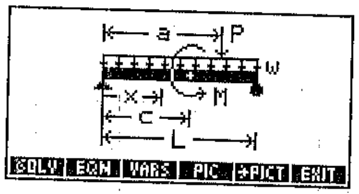
$$\sigma\max = \frac{P}{A} \cdot \left(1 + \frac{\epsilon \cdot c}{r^2} \cdot \left(\frac{1}{\cos\left(\frac{K \cdot L}{2 \cdot r} \cdot \sqrt{\frac{P}{E \cdot A}}\right)}\right)\right) \qquad r = \sqrt{\frac{I}{A}}$$

Example:

Given: L=6.6542_m, A=187.9351_cm^2, r=8.4836_cm, E=206842718.795_kPa, I=135259652.16_mm^4, K=1, P=1908.2571_kN, c=15.24_cm, ϵ=1.1806_cm.

Solution: σmax=140853.0970_kPa.

Simple Deflection (1, 3)



Equation:

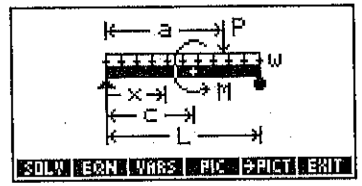
$$y = \frac{P \cdot (L-a) \cdot x}{6 \cdot L \cdot E \cdot I} \cdot \left(x^2 + (L-a)^2 - L^2\right) - \frac{M \cdot x}{E \cdot I} \cdot \left(c - \frac{x^2}{6 \cdot L} - \frac{L}{3} - \frac{c^2}{2 \cdot L}\right) - \frac{w \cdot x}{24 \cdot E \cdot I} \cdot \left(L^3 + x^2 \cdot (x - 2 \cdot L)\right)$$

Example:

Given: L=20_ft, E=29000000_psi, I=40_in^4, a=10_ft, P=674.427_lbf, c=17_ft, M=3687.81_ft*lbf, w=102.783_lbf/ft, x=9_ft.

Solution: y=–.6005_in.

Simple Slope (1, 4)



Equation:

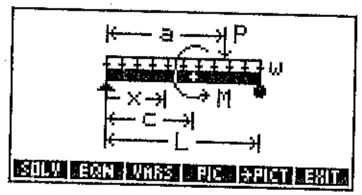
$$\Theta = \frac{P \cdot (L-a)}{6 \cdot L \cdot E \cdot I} \cdot \left(3 \cdot x^2 + (L-a)^2 - L^2\right) - \frac{M}{E \cdot I} \cdot \left(c - \frac{x^2}{2 \cdot L} - \frac{L}{3} - \frac{c^2}{2 \cdot L}\right) - \frac{w}{24 \cdot E \cdot I} \cdot \left(L^3 + x^2 \cdot (4 \cdot x - 6 \cdot L)\right)$$

Example:

Given: L=20_ft, E=29000000_psi, I=40_in^4, a=10_ft, P=674.427_lbf, c=17_ft, M=3687.81_ft*lbf, w=102.783_lbf/ft, x=9_ft.

Solution: Θ=–.0876_°

Simple Moment (1, 5)



Equation:

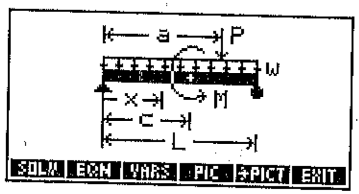
$$Mx = \frac{P \cdot (L - a) \cdot x}{L} + \frac{M \cdot x}{L} + \frac{w \cdot x}{2} \cdot (L - x)$$

Example:

Given: L=20_ft, a=10_ft, P=674.427_lbf, c=17_ft, M=3687.81_ft*lbf, w=102.783_lbf/ft, x=9_ft.

Solution: Mx=9782.1945_ft*lbf.

Simple Shear (1, 6)



Equation:

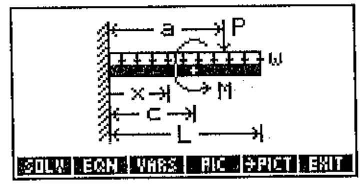
$$V = \frac{P \cdot (L - a)}{L} + \frac{M}{L} + \frac{w}{2} \cdot (L - 2 \cdot x)$$

Example:

Given: L=20_ft, a=10_ft, P=674.427_lbf, M=3687.81_ft*lbf, w=102.783_lbf/ft, x=9_ft.

Solution: V=624.387_lbf.

Cantilever Deflection (1, 7)



Equation:

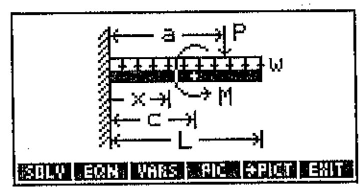
$$y = \frac{P \cdot x^2}{6 \cdot E \cdot I} \cdot (x - 3 \cdot a) + \frac{M \cdot x^2}{2 \cdot E \cdot I} - \frac{w \cdot x^2}{24 \cdot E \cdot I} \cdot (6 \cdot L^2 - 4 \cdot L \cdot x + x^2)$$

Example:

Given: L=10_ft, E=29000000_psi, I=15_in^4, P=500_lbf, M=800_ft*lbf, a=3_ft, c=6_ft, w=100_lbf/ft, x=8_ft.

Solution: y=−.3316_in.

Cantilever Slope (1, 8)



Equation:

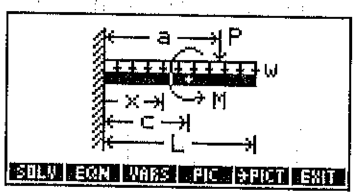
$$\Theta = \frac{P\cdot x}{2\cdot E\cdot I}\cdot\left(x - 2\cdot a\right) + \frac{M\cdot x}{E\cdot I} - \frac{w\cdot x}{6\cdot E\cdot I}\cdot\left(3\cdot L^2 - 3\cdot L\cdot x + x^2\right)$$

Example:

Given: L=10_ft, E=29000000_psi, I=15_in^4, P=500_lbf, M=800_ft*lbf, a=3_ft, c=6_ft, w=100_lbf/ft, x=8_ft.

Solution: Θ=–.2652_°

Cantilever Moment (1, 9)



Equation:

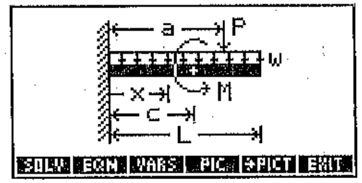
$$Mx = P\cdot\left(x - a\right) + M - \frac{W}{2}\cdot\left(L^2 - 2\cdot L\cdot x + x^2\right)$$

Example:

Given: L=10_ft, P=500_lbf, M=800_ft*lbf, a=3_ft, c=6_ft, w=100_lbf/ft, x=8_ft.

Solution: Mx=–200_ft*lbf.

Cantilever Shear (1, 10)



Equation:

$$V = P + w\cdot\left(L - x\right)$$

Example:

Given: L=10_ft, P=500_lbf, a=3_ft, x=8_ft, w=100_lbf/ft.

Solution: V=200_lbf.

Electricity (2)

Variable Names and Descriptions

ϵr	Relative permittivity
μr	Relative permeability
ω	Angular frequency
$\omega 0$	Resonant angular frequency
ϕ	Phase angle
ϕp,ϕs	Parallel and series phase angles
ρ	Resistivity

Variable Names and Descriptions (continued)

ΔI	Current change
Δt	Time change
ΔV	Voltage change
A	Wire cross-section area (Wire Resistance), or Solenoid cross-section area (Solenoid Inductance), or Plate area (Plate Capacitor)
$C, C1, C2$	Capacitance
Cp, Cs	Parallel and series capacitances
d	Plate separation
E	Energy
F	Force between charges
f	Frequency
$f0$	Resonant frequency
I	Current, or Total current (Current Divider)
$I1$	Current in R1
$Imax$	Maximum current
L	Inductance, or Length (Wire Resistance, Cylindrical Capacitor)
$L1, L2$	Inductance
Lp, Ls	Parallel and series inductances
N	Number of turns
n	Number of turns per unit length
P	Power
q	Charge
$q1, q2$	Point charge
Qp, Qs	Parallel and series quality factors
r	Charge distance
$R, R1, R2$	Resistance
ri, ro	Inside and outside radii
Rp, Rs	Parallel and series resistances

Variable Names and Descriptions (continued)

t	Time
ti, tf	Initial and final times
V	Voltage, or Total voltage (Voltage Divider)
$V1$	Voltage across R1
Vi, Vf	Initial and final voltages
$Vmax$	Maximum voltage
XC	Reactance of capacitor
XL	Reactance of inductor

Reference: 3.

Coulomb's Law (2, 1)

This equation describes the electrostatic force between two charged particles.

Equation:

$$\mathrm{F} = \frac{1}{4 \cdot \pi \cdot \epsilon 0 \cdot \epsilon \mathrm{r}} \cdot \left(\frac{\mathrm{q1} \cdot \mathrm{q2}}{\mathrm{r}^2} \right)$$

Example:

Given: $q1$=1.6E−19_C, $q2$=1.6E−19_C, r=4.00E−13_cm, ϵr=1.00.

Solution: F=14.3801_N.

Ohm's Law and Power (2, 2)

Equations:

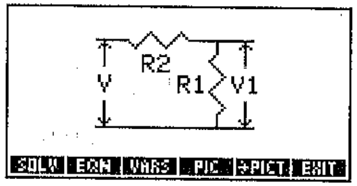
$$\mathrm{V} = \mathrm{I} \cdot \mathrm{R} \qquad \mathrm{P} = \mathrm{V} \cdot \mathrm{I} \qquad \mathrm{P} = \mathrm{I}^2 \cdot \mathrm{R} \qquad \mathrm{P} = \frac{\mathrm{V}^2}{\mathrm{R}}$$

Example:

Given: V=24_V, I=16_A.

Solution: R=1.5_Ω, P=384_W.

Voltage Divider (2, 3)



Equation:

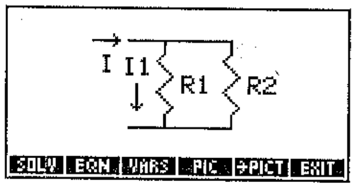
$$V1 = V \cdot \left(\frac{R1}{R1 + R2} \right)$$

Example:

Given: $R1$=40_Ω, $R2$=10_Ω, V=100_V.

Solution: $V1$=80_V.

Current Divider (2, 4)



Equation:

$$I1 = I \cdot \left(\frac{R2}{R1 + R2} \right)$$

Example:

Given: $R1$=10_Ω, $R2$=6_Ω, I=15_A.

Solution: $I1$=5.6250_A.

Wire Resistance (2, 5)

Equation:

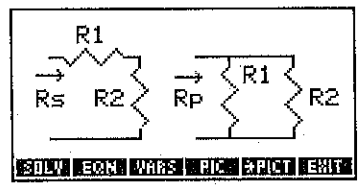
$$R = \frac{\rho \cdot L}{A}$$

Example:

Given: ρ=.0035_Ω*cm, L=50_cm, A=1_cm^2.

Solution: R=0.175_Ω.

Series and Parallel R (2, 6)



Equations:

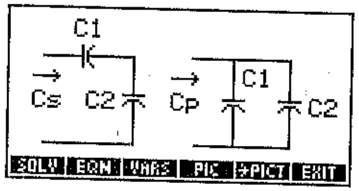
$$Rs = R1 + R2 \qquad \frac{1}{Rp} = \frac{1}{R1} + \frac{1}{R2}$$

Example:

Given: $R1$=2_Ω, $R2$=3_Ω.

Solution: Rs=5_Ω, Rp=1.2000_Ω.

Series and Parallel C (2, 7)



Equations:

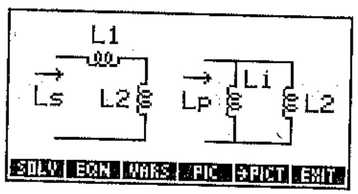
$$\frac{1}{\mathrm{Cs}} = \frac{1}{\mathrm{C1}} + \frac{1}{\mathrm{C2}} \qquad \mathrm{Cp} = \mathrm{C1} + \mathrm{C2}$$

Example:

Given: $C1$=2_μF, $C2$=3_μF.

Solution: Cs=1.2000_μF, Cp=5_μF.

Series and Parallel L (2, 8)



Equations:

$$\mathrm{Ls} = \mathrm{L1} + \mathrm{L2} \qquad \frac{1}{\mathrm{Lp}} = \frac{1}{\mathrm{L1}} + \frac{1}{\mathrm{L2}}$$

Example:

Given: $L1$=17_mH, $L2$=16.5_mH.

Solution: Ls=33.5000_mH, Lp=8.3731_mH.

Capacitive Energy (2, 9)

Equation:

$$\mathrm{E} = \frac{\mathrm{C} \cdot \mathrm{V}^2}{2}$$

Example:

Given: E=.025_J, C=20_μF.

Solution: V=50_V.

Inductive Energy (2, 10)

Equation:

$$\mathrm{E} = \frac{\mathrm{L} \cdot \mathrm{I}^2}{2}$$

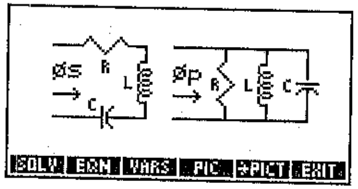
Example:

Given: E=4_J, L=15_mH.

Solution: I=23.0940_A.

RLC Current Delay (2, 11)

The phase delay (angle) is positive for current lagging voltage.



Equations:

$$\mathrm{TAN}\left(\phi s\right) = \frac{XL - XC}{R} \qquad \mathrm{TAN}\left(\phi p\right) = \frac{\frac{1}{XC} - \frac{1}{XL}}{\frac{1}{R}}$$

$$XC = \frac{1}{\omega \cdot C} \qquad XL = \omega \cdot L \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: f=107_Hz, C=80_μF, L=20_mH, R=5_Ω.

Solution: ω=672.3008_r/s, ϕs=−45.8292_°, ϕp=−5.8772_°, XC=18.5929_Ω, XL=13.4460_Ω.

DC Capacitor Current (2, 12)

These equations approximate the dc current required to change the voltage on a capacitor in a certain time interval.

Equations:

$$I = C \cdot \left(\frac{\Delta V}{\Delta t}\right) \qquad \Delta V = Vf - Vi \qquad \Delta t = tf \cdot ti$$

Example:

Given: C=15_μF, Vi=2.3_V, Vf=3.2_V, I=10_A, ti=0_s.

Solution: ΔV=.9000_V, Δt=1.3500_μs, tf=1.3500_μs.

Capacitor Charge (2, 13)

Equation:

$$q = C \cdot V$$

Example:

Given: C=20_μF, V=100_V.

Solution: q=0.0020_C.

DC Inductor Voltage (2, 14)

These equations approximate the dc voltage induced in an inductor by a change in current in a certain time interval.

Equations:

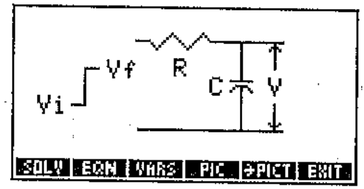
$$V = -L \cdot \left(\frac{\Delta I}{\Delta t}\right) \qquad \Delta I = If - Ii \qquad \Delta t = tf - ti$$

Example:

Given: L=100_mH, V=52_V, Δt=32_μs, Ii=23_A, ti=0_s.

Solution: ΔI=−0.0166_A, If=22.9834_A, tf=32_μs.

RC Transient (2, 15)



Equation:

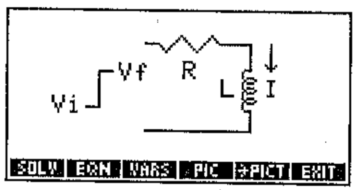
$$V = Vf - \left(Vf - Vi\right) \cdot e^{\frac{-t}{R \cdot C}}$$

Example:

Given: Vi=0_V, C=50_μF, Vf=10_V, R=100_ω, t=2_ms.

Solution: V=3.2968_V.

RL Transient (2, 16)



Equation:

$$I = \frac{1}{R} \cdot \left(Vf - \left(Vf - Vi\right) \cdot e^{\frac{-t \cdot R}{L}}\right)$$

Example:

Given: Vi=0_V, Vf=5_V, R=50_ω, L=50_mH, t=75_μs.

Solution: I=0.0072_A.

Resonant Frequency (2, 17)

Equations:

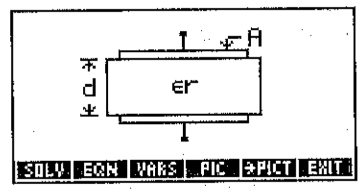
$$\omega 0 = \frac{1}{\sqrt{L \cdot C}} \qquad Qs = \frac{1}{R} \cdot \sqrt{\frac{L}{C}} \qquad Qp = R \cdot \sqrt{\frac{C}{L}} \qquad \omega 0 = 2 \cdot \pi \cdot f0$$

Example:

Given: L=500_mH, C=8_μF, R=10_ω.

Solution: $\omega 0$=500_r/s, Qs=25.0000, Qp=0.0400, $f0$=79.5775_Hz.

Plate Capacitor (2, 18)



Equation:

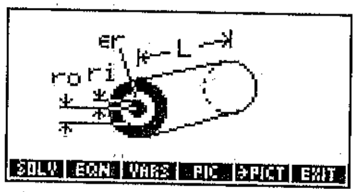
$$C = \frac{\epsilon 0 \cdot \epsilon r \cdot A}{d}$$

Example:

Given: C=25_μF, ϵr=2.26, A=1_cm^2.

Solution: d=8.0042E−9_cm.

Cylindrical Capacitor (2, 19)



Equation:

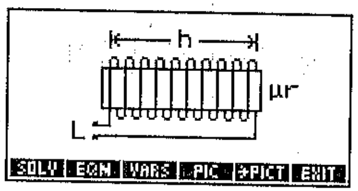
$$C = \frac{2\cdot\pi\cdot\epsilon 0\cdot\epsilon r\cdot L}{LN\left(\frac{ro}{ri}\right)}$$

Example:

Given: ϵr=1, ro=1_cm, ri=.999_cm, L=10_cm.

Solution: C=0.0056_μF.

Solenoid Inductance (2, 20)



Equation:

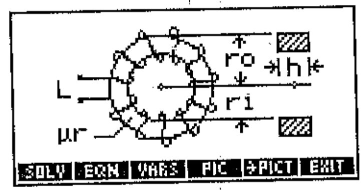
$$L = \mu 0\cdot\mu r\cdot n^2\cdot A\cdot h$$

Example:

Given: μr=2.5, n=40_1/cm, A=.2_cm^2, h=3_cm.

Solution: L=0.0302_mH.

Toroid Inductance (2, 21)



Equation:

$$L = \frac{\mu 0\cdot\mu r\cdot N^2\cdot h}{2\cdot\pi}\cdot LN\left(\frac{ro}{ri}\right)$$

Example:

Given: μr=1, N=5000, h=2_cm, ri=2_cm, ro=4_cm.

Solution: L=69.3147_mH.

Sinusoidal Voltage (2, 22)

Equations:

$$V = Vmax\cdot SIN\left(\omega\cdot t+\phi\right) \qquad \omega = 2\cdot\pi\cdot f$$

Example:

Given: $Vmax$=110_V, t=30_μs, f=60_Hz, ϕ=15_°

Solution: ω=376.9911_r/s, V=29.6699_V.

Sinusoidal Current (2, 23)

Equations:

$$I = Imax\cdot SIN\left(\omega\cdot t+\phi\right) \qquad \omega = 2\cdot\pi\cdot f$$

Example:

Given: t=32_s, $Imax$=10_A, ω=636_r/s, ϕ=30_°.

Solution: I=9.5983_A, f=101.2225_Hz.

Fluids (3)

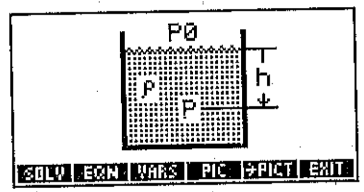
Variable Names and Descriptions

ϵ	Roughness
μ	Dynamic viscosity
ρ	Density
ΔP	Pressure change
Δy	Height change
ΣK	Total fitting coefficients
A	Cross-sectional area
$A1, A2$	Initial and final cross-sectional areas
D	Diameter
$D1, D2$	Initial and final diameters
h	Depth relative to $P0$ reference depth
hL	Head loss
L	Length
M	Mass flow rate
n	Kinematic viscosity
P	Pressure at h
$P0$	Reference pressure
$P1, P2$	Initial and final pressures
Q	Volume flow rate
Re	Reynolds number
$v1, v2$	Initial and final velocities
$vavg$	Average velocity
W	Power input
$y1, y2$	Initial and final heights

References: 3, 6, 9.

Pressure at Depth (3, 1)

This equation describes hydrostatic pressure for an incompressible fluid. Depth h is positive downward from the reference.



Equation:

$$P = P0 + \rho \cdot g \cdot h$$

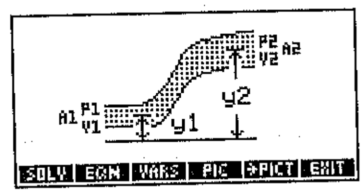
Example:

Given: h=100_m, ρ=1025.1817_kg/m^3, $P0$=1_atm.

Solution: P=1106.6848_kPa.

Bernoulli Equation (3, 2)

These equations represent the streamlined flow of an incompressible fluid.



Equations:

$$\frac{\Delta P}{\rho}+\frac{v2^2-v1^2}{2}+g\cdot\Delta y=0$$

$$\frac{\Delta P}{\rho}+\frac{v2^2\left(1-\left(\frac{A2}{A1}\right)^2\right)}{2}+g\cdot\Delta y=0$$

$$\frac{\Delta P}{\rho}+\frac{v1^2\left(\left(\frac{A1}{A2}\right)^2-1\right)}{2}+g\cdot\Delta y=0$$

$$\Delta P=P2-P1 \qquad \Delta y=y2-y1 \qquad M=\rho\cdot Q$$

$$Q=A2\cdot v2 \qquad Q=A1\cdot v1$$

$$A1=\frac{\pi\cdot D1^2}{4} \qquad A2=\frac{\pi\cdot D2^2}{4}$$

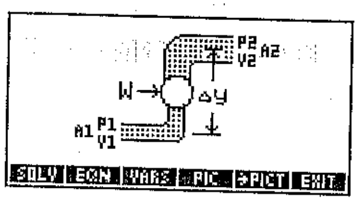
Example:

Given: $P2$=25_psi, $P1$=75_psi, $y2$=35_ft, $y1$=0_ft, $D1$=18_in, ρ=64_lb/ft^3, $v1$=100_ft/s.

Solution: Q=10602.8752_ft^3/min, M=678584.0132_lb/min, $v2$=122.4213_ft/s, $A2$=207.8633_in^2, $D2$=16.2684_in, $A1$=254.4690_in^2, ΔP=–50_psi, Δy=35_ft.

Flow with Losses (3, 3)

These equations extend Bernoulli's equation to include power input (or output) and head loss.



Equations:

$$M\cdot\left(\frac{\Delta P}{\rho}+\frac{v2^2-v1^2}{2}+g\cdot\Delta y+hL\right)=W$$

$$M\left(\frac{\Delta P}{\rho}+\frac{v2^2\left(1-\left(\frac{A2}{A1}\right)^2\right)}{2}+g\cdot\Delta y+hL\right)=W$$

$$M\left(\frac{\Delta P}{\rho}+\frac{v1^2\left(\left(\frac{A1}{A2}\right)^2-1\right)}{2}+g\cdot\Delta y+hL\right)=W$$

$$\Delta P=P2-P1 \qquad \Delta y=y2-y1 \qquad M=\rho\cdot Q$$

$$Q=A2\cdot v2 \qquad Q=A1\cdot v1$$

$$A1=\frac{\pi\cdot D1^2}{4} \qquad A2=\frac{\pi\cdot D2^2}{4}$$

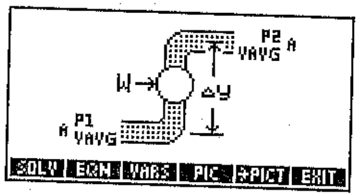
Example:

Given: $P2$=30_psi, $P1$=65_psi, $y2$=100_ft, $y1$=0_ft, ρ=64_lb/ft^3, $D1$=24_in, hL=2.0_ft^2/s^2, W=25_hp, $v1$=100_ft/s.

Solution: Q=18849.5559_ft^3/min, M=1206371.5790_lb/min, ΔP=–35_psi, Δy=100_ft, $v2$=93.1269_ft/s, $A1$=452.3893_in^2, $A2$=485.7773_in^2, $D2$=24.8699_in.

Flow in Full Pipes (3, 4)

These equations adapt Bernoulli's equation for flow in a round, full pipe, including power input (or output) and frictional losses. (See "FANNING" in chapter 3.)



Equations:

$$\rho \cdot \left(\frac{\pi \cdot D^2}{4}\right) \cdot vavg \cdot \left(\frac{\Delta P}{\rho} + g \cdot \Delta y + vavg^2 \cdot \left(2 \cdot f \cdot \left(\frac{L}{D}\right) + \frac{\Sigma K}{2}\right)\right) = W$$

$$\Delta P = P2 - P1 \qquad \Delta y = y2 - y1 \qquad M = \rho \cdot Q$$

$$Q = A \cdot vavg \qquad A = \frac{\pi \cdot D^2}{4} \qquad Re = \frac{D \cdot vavg \cdot \rho}{\mu} \qquad n = \frac{\mu}{\rho}$$

Example:

Given: ρ=62.4_lb/ft^3, *D*=12_in, *vavg*=8_ft/s, *P2*=15_psi, *P1*=20_psi, *y2*=40_ft, *y1*=0_ft, μ=.00002_lbf*s/ft^2, ΣK=2.25, ϵ=.02_in, *L*=250_ft.

Solution: ΔP=–5_psi, Δy=40_ft, *A*=113.0973_in^2, *n*=1.0312_ft^2/s, *Q*=376.9911_ft^3/min, *M*=23524.2458_lb/min, *W*=25.8897_hp, *Re*=775780.5.

Forces and Energy (4)

Variable Names and Descriptions

α	Angular acceleration
ω	Angular velocity
$\omega i, \omega f$	Initial and final angular velocities
ρ	Fluid density
τ	Torque
Θ	Angular displacement
a	Acceleration
A	Projected area relative to flow
ar	Centripetal acceleration at r
at	Tangential acceleration at r
Cd	Drag coefficient
E	Energy
F	Force at r or x, or Spring force (Hooke's Law), or Attractive force (Law of Gravitation), or Drag force (Drag Force)
I	Moment of inertia
k	Spring constant
Ki, Kf	Initial and final kinetic energies
$m, m1, m2$	Mass
N	Rotational speed
Ni, Nf	Initial and final rotational speeds
P	Instantaneous power
$Pavg$	Average power

Variable Names and Descriptions (continued)

r	Radius from rotation axis, or Separation distance (Law of Gravitation)
t	Time
v	Velocity
$vf, v1f, v2f$	Final velocity
$vi, v1i$	Initial velocity
W	Work
x	Displacement

Reference: 3.

Linear Mechanics (4, 1)

Equations:

$$F = m \cdot a \qquad Ki = \frac{1}{2} \cdot m \cdot vi^2 \qquad Kf = \frac{1}{2} \cdot m \cdot vf^2 \qquad W = F \cdot x$$

$$W = Kf - Ki \qquad P = F \cdot v \qquad Pavg = \frac{W}{t} \qquad vf = vi + a \cdot t$$

Example:

Given: t=10_s, m=50_lb, a=12.5_ft/s^2, vi=0_ft/s.

Solution: vf=125_ft/s, x=625_ft, F=19.4256_lbf, Ki=0_ft*lbf, Kf=12140.9961_ft*lbf, W=12140.9961_ft*lbf, $Pavg$=2.2075_hp.

Angular Mechanics (4, 2)

Equations:

$$\tau = I \cdot \alpha \qquad Ki = \frac{1}{2} \cdot I \cdot \omega i^2 \qquad Kf = \frac{1}{2} \cdot I \cdot \omega f^2 \qquad W = \tau \cdot \Theta$$

$$W = Kf - Ki \qquad P = \tau \cdot \omega \qquad Pavg = \frac{W}{t} \qquad \omega f = \omega i + \alpha \cdot t$$

$$at = \alpha \cdot r \qquad \omega = 2 \cdot \pi \cdot N \qquad \omega i = 2 \cdot \pi \cdot Ni \qquad \omega f = 2 \cdot \pi \cdot Nf$$

Example:

Given: I=1750_lb*in^2, Θ=360_°, r=3.5_in, α=10.5_r/min^2, ωi=0_r/s.

Solution: τ=1.1017E−3_ft*lbf, Ki=0_ft*lbf, W=6.9221E−3_ft*lbf, Kf=6.9221E−3_ft*lbf, at=8.5069E−4_ft/s^2, Ni=0_rpm, ωf=11.4868_r/min, t=1.0940_min, Nf=1.8282_rpm, $Pavg$=1.9174E−7_hp.

Centripetal Force (4, 3)

Equations:

$$F = m \cdot \omega^2 \cdot r \qquad \omega = \frac{v}{r} \qquad ar = \frac{v^2}{r} \qquad \omega = 2 \cdot \pi \cdot N$$

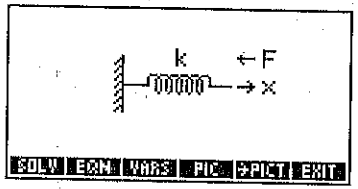
Example:

Given: m=1_kg, r=5_cm, N=2000_Hz.

Solution: ω=12566.3706_r/s, ar=7895683.5209_m/s, F=7895683.5209_N, v=628.3185_m/s.

Hooke's Law (4, 4)

The force is that exerted by the spring.



Equations:

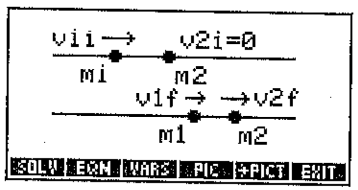
$$F = -k \cdot x \qquad W = \frac{-1}{2} \cdot k \cdot x^2$$

Example:

Given: k=1725_lbf/in, x=1.25_in.

Solution: F=−2156.25_lbf, W=−112.3047_ft*lbf.

1D Elastic Collisions (4, 5)



Equations:

$$v1f = \frac{m1 - m2}{m1 + m2} \cdot v1i \qquad v2f = \frac{2 \cdot m1}{m1 + m2} \cdot v1i$$

Example:

Given: $m1$=10_kg, $m2$=25_kg, $v1i$=100_m/s.

Solution: $v1f$=−42.8571_m/s, $v2f$=57.1429_m/s.

Drag Force (4, 6)

Equation:

$$F = Cd \cdot \left(\frac{\rho \cdot v^2}{2}\right) \cdot A$$

Example:

Given: Cd=.05, ρ=1000_kg/m^3, A=7.5E6_cm^2, v=35_m/s.

Solution: F=22968750_N.

Law of Gravitation (4, 7)

Equation:

$$F = G \cdot \left(\frac{m1 \cdot m2}{r^2}\right)$$

Example:

Given: $m1$=2E15_kg, $m2$=2E18_kg, r=1000000_km.

Solution: F=266903.6_N.

Mass-Energy Relation (4, 8)

Equation:

$$E = m \cdot c^2$$

Example:

Given: m=9.1E−31_kg.

Solution: E=8.1787E−14_J.

Gases (5)

Variable Names and Descriptions

λ	Mean free path
ρ	Flow density
$\rho 0$	Stagnation density
A	Flow area
At	Throat area
d	Molecular diameter
k	Specific heat ratio
M	Mach number
m	Mass
MW	Molecular weight
n	Number of moles, or Polytropic constant (Polytropic Processes)
P	Pressure, or Flow pressure (Isentropic Flow)
$P0$	Stagnation pressure
Pc	Pseudocritical pressure
Pi, Pf	Initial and final pressures
T	Temperature, or Flow temperature (Isentropic Flow)

Variable Names and Descriptions (continued)

$T0$	Stagnation temperature
Tc	Pseudocritical temperature
Ti, Tf	Initial and final temperatures
V	Volume
Vi, Vf	Initial and final volumes
$vrms$	Root-mean-square (rms) velocity
W	Work

References: 1, 3.

Ideal Gas Law (5, 1)

Equations:

$$\mathrm{P \cdot V = n \cdot R \cdot T} \qquad \mathrm{m = n \cdot MW}$$

Example:

Given: T=16.85_°C, P=1_atm, V=25_l, MW=36_g/gmol.

Solution: n=1.0506_gmol, m=3.7820E−2_kg.

Ideal Gas State Change (5, 2)

Equation:

$$\frac{\mathrm{Pf \cdot Vf}}{\mathrm{Tf}} = \frac{\mathrm{Pi \cdot Vi}}{\mathrm{Ti}}$$

Example:

Given: Pi=1.5_kPa, Pf=1.5_kPa, Vi=2_l, Ti=100_°C, Tf=373.15_K.

Solution: Vf=2_l.

Isothermal Expansion (5, 3)

These equations apply to an ideal gas.

Equations:

$$W = n \cdot R \cdot T \cdot LN\left(\frac{Vf}{Vi}\right) \qquad m = n \cdot MW$$

Example:

Given: Vi=2_l, Vf=125_l, T=300_°C, n=0.25_gmol, MW=64_g/gmol.

Solution: W=4926.4942_J, m=.016_kg.

Polytropic Processes (5, 4)

These equations describe a reversible pressure-volume change of an ideal gas such that $P * V^n$ is constant. Special cases include isothermal processes (n=1), isentropic processes (n=k, the specific heat ratio), and constant-pressure processes (n=0).

Equations:

$$\frac{Pf}{Pi} = \left(\frac{Vf}{Vi}\right)^{-n} \qquad \frac{Tf}{Ti} = \left(\frac{Pf}{Pi}\right)^{\frac{n-1}{n}}$$

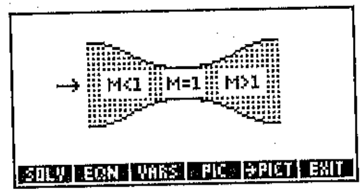
Example:

Given: Pi=15_psi, Pf=35_psi, Vi=1_ft^3, Vf=0.50_ft^3, Ti=75_°F.

Solution: n=1.2224, Tf=164.1117_°F.

Isentropic Flow (5, 5)

The calculation differs at velocities below and above Mach 1. The Mach number is based on the speed of sound in the compressible fluid.



Equations:

$$\frac{T}{T0} = \frac{2}{2+(k-1)\cdot M^2} \qquad \frac{P}{P0} = \left(\frac{T}{T0}\right)^{\frac{k}{k-1}}$$

$$\frac{\rho}{\rho 0} = \left(\frac{T}{T0}\right)^{\frac{1}{k-1}}$$

$$\frac{A}{At} = \frac{1}{M}\cdot\left(\frac{2}{k+1}\cdot\left(1+\frac{k-1}{2}\cdot M^2\right)\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Example:

Given: k=2, M=.9, $T0$=26.85_°C, T=373.15_K, $\rho 0$=100_kg/m^3, $P0$=100_kPa, A=1_cm^2.

Solution: P=464.1152_kPa, At=0.9928_cm^2, ρ=215.4333_kg/m^3.

Real Gas Law (5, 6)

These equations adapt the ideal gas law to emulate real-gas behavior. (See "ZFACTOR" in chapter 3.)

Equations:

$$\mathrm{P \cdot V = n \cdot Z \cdot R \cdot T} \qquad \mathrm{m = n \cdot MW}$$

Example:

Given: Pc=48_atm, Tc=298_K, P=5_kPa, V=10_l, MW=64_g/gmol, T=75_°C.

Solution: n=0.0173_gmol, m=1.1057E−3_kg.

Real Gas State Change (5, 7)

This equation adapts the ideal gas state-change equation to emulate real-gas behavior. (See "ZFACTOR" in chapter 3.)

Equation:

$$\mathrm{\frac{Pf \cdot Vf}{Zf \cdot Tf} = \frac{Pi \cdot Vi}{Zi \cdot Ti}}$$

Example:

Given: Pc=48_atm, Pi=100_kPa, Pf=50_kPa, Ti=75_°C, Tc=298_K, Vi=10_l, Tf=250_°C.

(Remember Zf and Zi are automatically calculated by the HP 48 using these variables.)

Solution: Vf=30.1703_l.

Kinetic Theory (5, 8)

These equations describe properties of an ideal gas.

Equations:

$$\mathrm{P = \frac{n \cdot MW \cdot vrms^2}{3 \cdot V}} \qquad \mathrm{vrms = \sqrt{\frac{3 \cdot R \cdot T}{MW}}}$$

$$\lambda = \frac{1}{\sqrt{2} \cdot \pi \cdot \left(\frac{\mathrm{n \cdot NA}}{\mathrm{V}}\right) \cdot \mathrm{d}^2} \qquad \mathrm{m = n \cdot MW}$$

Example:

Given: P=100_kPa, V=2_l, T=26.85_°C, MW=18_g/gmol, d=2.5_nm.

Solution: $vrms$=644.7678_m/s, m=1.4433E−3_kg, n=.0802_gmol, λ=1.4916_nm.

Heat Transfer (6)

Variable Names and Descriptions

α	Expansion coefficient
δ	Elongation
$\lambda 1, \lambda 2$	Lower and upper wavelength limits
λmax	Wavelength of maximum emissive power
ΔT	Temperature difference
A	Area
c	Specific heat

Variable Names and Descriptions (continued)

eb12	Emissive power in the range $\lambda 1$ to $\lambda 2$
eb	Total emissive power
f	Fraction of emissive power in the range $\lambda 1$ to $\lambda 2$
h,h1,h3	Convective heat-transfer coefficient
k,k1,k2,k3	Thermal conductivity
L,L1,L2,L3	Length
m	Mass
Q	Heat capacity
q	Heat transfer rate
T	Temperature
Tc	Cold surface temperature (Conduction), or Cold fluid temperature
Th	Hot surface temperature, or Hot fluid temperature (Conduction + Convection)
Ti,Tf	Initial and final temperatures
U	Overall heat transfer coefficient

References: 7, 9.

Heat Capacity (6, 1)

Equations:

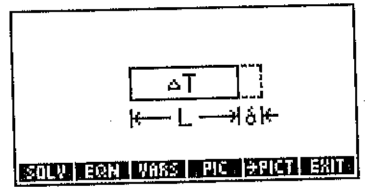
$$Q = m \cdot c \cdot \Delta T \qquad Q = m \cdot c \cdot \left(Tf - Ti\right)$$

Example:

Given: ΔT=15_°C, Ti=0_°C, m=10_kg, Q=25_kJ.

Solution: Tf=15_°C, c=.1667_kJ/(kg*K).

Thermal Expansion (6, 2)



Equations:

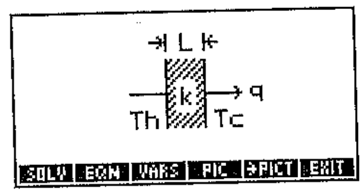
$$\delta = \alpha \cdot L \cdot \Delta T \qquad \delta = \alpha \cdot L \cdot \left(Tf - Ti\right)$$

Example:

Given: ΔT=15_°C, L=10_m, Tf=25_°C, δ=1_cm.

Solution: Ti=10_°C, α=6.6667E−5_1/°C.

Conduction (6, 3)



Equations:

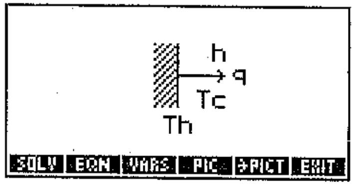
$$q = \frac{k \cdot A}{L} \cdot \Delta T \qquad q = \frac{k \cdot A}{L} \cdot \left(Th - Tc\right)$$

Example:

Given: Tc=25_°C, Th=75_°C, A=12.5_m^2, L=1.5_cm, k=.12_W/(m*K).

Solution: q=5000_W, ΔT=50_°C.

Convection (6, 4)



Equations:

$$q = h \cdot A \cdot \Delta T \qquad q = h \cdot A \cdot \left(Th - Tc \right)$$

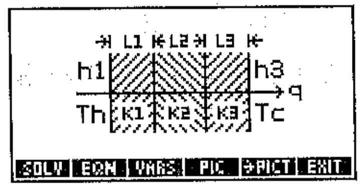
Example:

Given: Tc=300_K, A=200_m^2, h=.005_W/(m^2*K), q=10_W.

Solution: ΔT=10_°C, Th=36.8500_°C.

Conduction + Convection (6, 5)

If you have fewer than three layers, give the extra layers a zero thickness and any nonzero conductivity. The two temperatures are fluid temperatures—if instead you know a *surface* temperature, set the corresponding convective coefficient to 10^{499}



Equations:

$$q = \frac{A \cdot \Delta T}{\frac{1}{h1} + \frac{L1}{k1} + \frac{L2}{k2} + \frac{L3}{k3} + \frac{1}{h3}} \qquad q = \frac{A \cdot \left(Th - Tc \right)}{\frac{1}{h1} + \frac{L1}{k1} + \frac{L2}{k2} + \frac{L3}{k3} + \frac{1}{h3}}$$

$$U = \frac{q}{A \cdot \Delta T} \qquad U = \frac{q}{A \cdot \left(Th - Tc \right)}$$

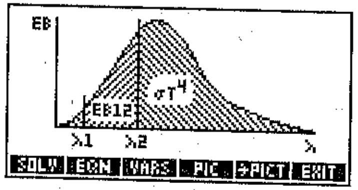
Example:

Given: ΔT=35_°C, Th=55_°C, A=10_m^2, $h1$=.05_W/(m^2*K), $h3$=.05_W/(m^2*K), $L1$=3_cm, $L2$=5_cm, $L3$=3_cm, $k1$=.1_W/(m*K), $k2$=.5_W/(m*K), $k3$=.1_W/(m*K).

Solution: Tc=20_°C, U=0.0246_W/(m^2*K), q=8.5995_W.

Black Body Radiation (6, 6)

See “F0λ” in chapter 3.



Equations:

$$eb = \sigma \cdot T^4 \qquad f = F0\lambda\left(\lambda 2; T\right) - F0\lambda\left(\lambda 1; T\right)$$

$$eb12 = f \cdot eb \qquad \lambda max \cdot T = c3 \qquad q = eb \cdot A$$

Example:

Given: T=1000_°C, $\lambda 1$=1000_nm, $\lambda 2$=600_nm, A=1_cm^2.

Solution: λmax=2276.0523_nm, eb=148984.2703_W/m^2, f=.0036, $eb12$=537.7264_W/m^2, q=14.8984_W.

Magnetism (7)

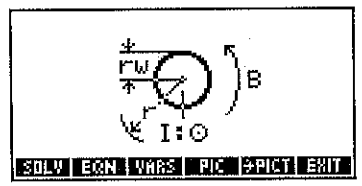
Variable Names and Descriptions

μr	Relative permeability
B	Magnetic field
d	Separation distance
Fba	Force
I, Ia, Ib	Current
L	Length
N	Total number of turns
n	Number of turns per unit length
r	Distance from center of wire
ri, ro	Inside and outside radii of toroid
rw	Radius of wire

Reference: 3.

Straight Wire (7, 1)

The magnetic field calculation differs depending upon whether the point is inside or outside the wire.



Equation:

$$B = \frac{\mu 0 \cdot \mu r \cdot I}{2 \cdot \pi \cdot r}$$

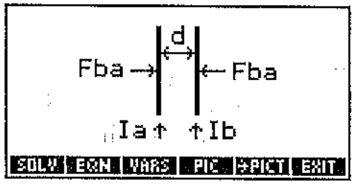
Example:

Given: μr=1, rw=.25_cm, r=.2_cm, I=25_A.

Solution: B=.0016_T.

Force between Wires (7, 2)

The force between wires is positive for an attractive force (for currents having the same sign).



Equation:

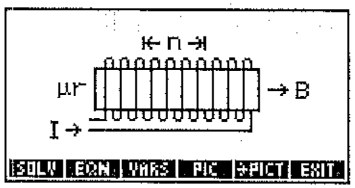
$$\mathrm{Fba} = \frac{\mu 0 \cdot \mu \mathrm{r} \cdot \mathrm{L} \cdot \mathrm{Ib} \cdot \mathrm{Ia}}{2 \cdot \pi \cdot \mathrm{d}}$$

Example:

Given: Ia=10_A, Ib=20_A, μr=1, L=50_cm, d=1_cm.

Solution: Fba=2.0000E−3_N.

Magnetic (B) Field in Solenoid (7, 3)



Equation:

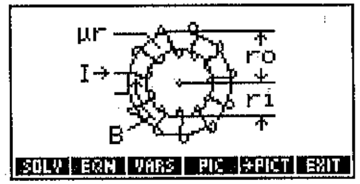
$$\mathrm{B} = \mu 0 \cdot \mu \mathrm{r} \cdot \mathrm{I} \cdot \mathrm{n}$$

Example:

Given: μr=10, n=50, I=1.25_A.

Solution: B=0.0785_T.

Magnetic (B) Field in Toroid (7, 4)



Equation:

$$\mathrm{B} = \frac{\mu 0 \cdot \mu \mathrm{r} \cdot \mathrm{I} \cdot \mathrm{N}}{2 \cdot \pi} \cdot \left(\frac{2}{\mathrm{ro} + \mathrm{ri}} \right)$$

Example:

Given: μr=10, N=50, ri=5_cm, ro=7_cm, I=10_A.

Solution: B=1.6667E−2_T.

Motion (8)

Variable Names and Descriptions

α	Angular acceleration
ω	Angular velocity (Circular Motion), or Angular velocity at t (Angular Motion)
$\omega 0$	Initial angular velocity
ρ	Fluid density
θ	Angular position at t
$\theta 0$	Initial angular position (Angular Motion), or Initial vertical angle (Projectile Motion)
a	Acceleration
A	Projected horizontal area
ar	Centripetal acceleration at r
Cd	Drag coefficient
m	Mass
M	Planet mass
N	Rotational speed
R	Horizontal range (Projectile Motion), or Planet radius (Escape Velocity)
r	Radius
t	Time
v	Velocity at t (Linear Motion), or Tangential velocity at r (Circular Motion), or Terminal velocity (Terminal Velocity), or Escape velocity (Escape Velocity)
$v0$	Initial velocity
vx	Horizontal component of velocity at t
vy	Vertical component of velocity at t
x	Horizontal position at t
$x0$	Initial horizontal position
y	Vertical position at t
$y0$	Initial vertical position

Reference: 3.

Linear Motion (8, 1)

Equations:

$$x = x0 + v0 \cdot t + \frac{1}{2} \cdot a \cdot t^2 \qquad x = x0 + v \cdot t - \frac{1}{2} \cdot a \cdot t^2$$

$$x = x0 + \frac{1}{2} \cdot \left(v0 + v\right) \cdot t \qquad v = v0 + a \cdot t$$

Example:

Given: $x0$=0_m, x=100_m, t=10_s, $v0$=1_m/s.

Solution: v=19_m/s, a=1.8_m/s^2.

Object in Free Fall (8, 2)

Equations:

$$y = y0 + v0 \cdot t - \frac{1}{2} \cdot g \cdot t^2 \qquad y = y0 + v \cdot t + \frac{1}{2} \cdot g \cdot t^2$$

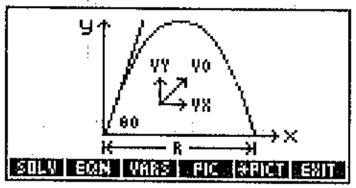
$$v^2 = v0^2 - 2 \cdot g \cdot \left(y - y0\right) \qquad v = v0 - g \cdot t$$

Example:

Given: $y0$=1000_ft, y=0_ft, $v0$=0_ft/s.

Solution: t=7.8843_s, v=−253.6991_ft/s.

Projectile Motion (8, 3)



Equations:

$$x = x0 + v0 \cdot COS\left(\theta 0\right) \cdot t \qquad y = y0 + v0 \cdot SIN\left(\theta 0\right) \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

$$vx = v0 \cdot COS\left(\theta 0\right) \qquad vy = v0 \cdot SIN\left(\theta 0\right) - g \cdot t$$

$$R = \frac{v0^2}{g} \cdot SIN\left(2 \cdot \theta 0\right)$$

Example:

Given: $x0$=0_ft, $y0$=0_ft, $\Theta 0$=45_°, $v0$=200_ft/s, t=10_s.

Solution: R=1243.2399_ft, vx=141.4214_ft/s, vy=−180.3186_ft/s, x=1414.2136_ft, y=−194.4864_ft.

Angular Motion (8, 4)

Equations:

$$\theta = \theta 0 + \omega 0 \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2 \qquad \theta = \theta 0 + \omega \cdot t - \frac{1}{2} \cdot \alpha \cdot t^2$$

$$\theta = \theta 0 + \frac{1}{2} \cdot \left(\omega 0 + \omega\right) \cdot t \qquad \omega = \omega 0 + \alpha \cdot t$$

Example:

Given: $\Theta 0$=0_°, $\omega 0$=0_r/min, α=1.5_r/min^2, t=30_s.

Solution: Θ=10.7430_°, ω=.7500_r/min.

Circular Motion (8, 5)

Equations:

$$\omega = \frac{v}{r} \qquad ar = \frac{v^2}{r} \qquad \omega = 2 \cdot \pi \cdot N$$

Example:

Given: r=25_in, v=2500_ft/s.

Solution: ω=72000_r/min, ar=3000000_ft/s^2, N=11459.1559_rpm.

Terminal Velocity (8, 6)

Equation:

$$v = \sqrt{\frac{2 \cdot m \cdot g}{Cd \cdot \rho \cdot A}}$$

Example:

Given: Cd=.15, ρ=.025lb/ft^3, A=100000_in^2, m=1250_lb.

Solution: v=1757.4709_ft/s.

Escape Velocity (8, 7)

Equation:

$$v = \sqrt{\frac{2 \cdot G \cdot M}{R}}$$

Example:

Given: M=1.5E23_lb, R=5000_mi.

Solution: v=3485.1106_ft/s.

Optics (9)

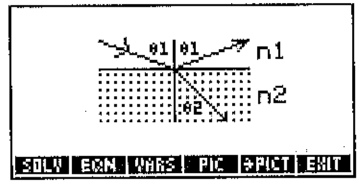
Variable Names and Descriptions

$\theta 1$	Angle of incidence
$\theta 2$	Angle of refraction
θB	Brewster angle
θc	Critical angle
f	Focal length
m	Magnification
$n, ni, n2$	Index of refraction
$r, r1, r2$	Radius of curvature
u	Distance to object
v	Distance to image

For reflection and refraction problems, the focal length and radius of curvature are positive in the direction of the outgoing light (reflected or refracted). The object distance is positive in front of the surface. The image distance is positive in the direction of the outgoing light (reflected or refracted). The magnification is positive for an upright image.

Reference: 3.

Law of Refraction (9, 1)



Equation:

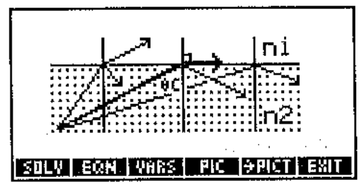
$$\mathrm{n1} \cdot \mathrm{SIN}\left(\theta 1\right) = \mathrm{n2} \cdot \mathrm{SIN}\left(\theta 2\right)$$

Example:

Given: $n1$=1, $n2$=1.333, $\theta 1$=45_°

Solution: $\theta 2$=32.0367_°

Critical Angle (9, 2)



Equation:

$$\mathrm{SIN}\left(\theta c\right) = \frac{\mathrm{n1}}{\mathrm{n2}}$$

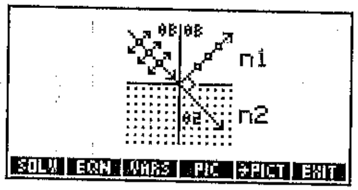
Example:

Given: $n1$=1, $n2$=1.5.

Solution: θc=41.8103_°

Brewster's Law (9, 3)

The Brewster angle is the angle of incidence at which the reflected wave is completely polarized.



Equations:

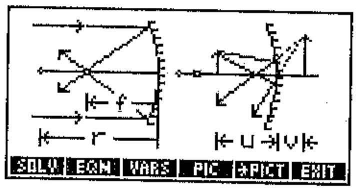
$$\mathrm{TAN}\left(\theta \mathrm{B}\right) = \frac{\mathrm{n2}}{\mathrm{n1}} \qquad \theta\mathrm{B} + \theta 2 = 90$$

Example:

Given: $n1$=1, $n2$=1.5.

Solution: θB=56.3099_°, $\theta 2$=33.6901_°

Spherical Reflection (9, 4)



Equations:

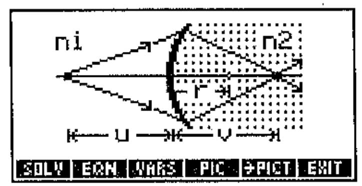
$$\frac{1}{\mathrm{u}} + \frac{1}{\mathrm{v}} = \frac{1}{\mathrm{f}} \qquad \mathrm{f} = \frac{1}{2}\cdot\mathrm{r} \qquad \mathrm{m} = \frac{-\mathrm{v}}{\mathrm{u}}$$

Example:

Given: u=10_cm, v=300_cm, r=19.35_cm.

Solution: m=−30, f=9.6774_cm.

Spherical Refraction (9, 5)



Equation:

$$\frac{\mathrm{n1}}{\mathrm{u}} + \frac{\mathrm{n2}}{\mathrm{v}} = \frac{\mathrm{n2}-\mathrm{n1}}{\mathrm{r}}$$

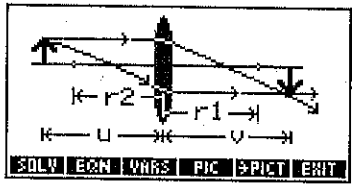
Example:

Given: u=8_cm, v=12_cm, r=2_cm, $n1$=1.

Solution: $n2$=1.5000.

Thin Lens (9, 6)

r1 is for the front surface, and *r2* is for the back surface.



Equations:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad \frac{1}{f} = \left(n - 1\right) \cdot \left(\frac{1}{r1} - \frac{1}{r2}\right) \qquad m = \frac{-v}{u}$$

Example:

Given: $r1$=5_cm, $r2$=20_cm, n=1.5, u=50_cm.

Solution: f=13.3333_cm, v=18.1818_cm, m=–.3636.

Oscillations (10)

Variable Names and Descriptions

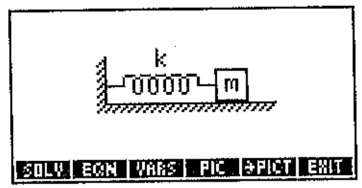
ω	Angular frequency
ϕ	Phase angle
θ	Cone angle
a	Acceleration at t
f	Frequency
G	Shear modulus of elasticity
h	Cone height

Variable Names and Descriptions (continued)

I	Moment of inertia
J	Polar moment of inertia
k	Spring constant
L	Length of pendulum
m	Mass
t	Time
T	Period
v	Velocity at t
x	Displacement at t
xm	Displacement amplitude

Reference: 3.

Mass-Spring System (10, 1)



Equations:

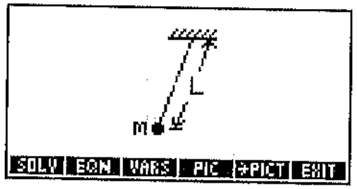
$$\omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\cdot\pi}{\omega} \qquad \omega = 2\cdot\pi\cdot f$$

Example:

Given: k=20_N/m, m=5_kg.

Solution: ω=2_r/s, T=3.1416_s, f=.3183_Hz.

Simple Pendulum (10, 2)



Equations:

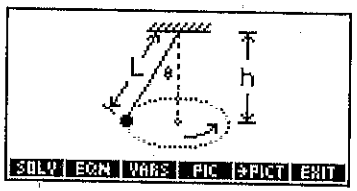
$$\omega = \sqrt{\frac{g}{L}} \qquad T = \frac{2 \cdot \pi}{\omega} \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: L=15_cm.

Solution: ω=8.0856_r/s, T=.7771_s, f=1.2869_Hz.

Conical Pendulum (10, 3)



Equations:

$$\omega = \sqrt{\frac{g}{h}} \qquad h = L \cdot COS\left(\theta\right) \qquad T = \frac{2 \cdot \pi}{\omega}$$

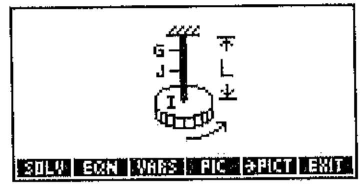
$$\omega = 2 \cdot \pi \cdot f$$

Example:

Given: L=25_cm, h=20_cm.

Solution: θ=36.899_°, T=.8973_s, ω=7.0024_r/s, f=1.1145_Hz.

Torsional Pendulum (10, 4)



Equations:

$$\omega = \sqrt{\frac{G \cdot J}{L \cdot I}} \qquad T = \frac{2 \cdot \pi}{\omega} \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: G=1000_kPa, J=17_mm^4, L=26_cm, I=50_kg*m^2.

Solution: ω=1.1435E−3_r/s, f=1.8200E−4_Hz, T=5494.4862_s.

Simple Harmonic (10, 5)

Equations:

$$x = xm \cdot COS\left(\omega \cdot t + \phi\right) \qquad v = -\omega \cdot xm \cdot SIN\left(\omega \cdot t + \phi\right)$$

$$a = -\omega^2 \cdot xm \cdot COS\left(\omega \cdot t + \phi\right) \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: xm=10_cm, ω=15_r/s, ϕ=25_°, t=25_μs.

Solution: x=9.0615_cm, v=−0.6344_m/s, a=−20.3884_m/s^2, f=2.3873_Hz.

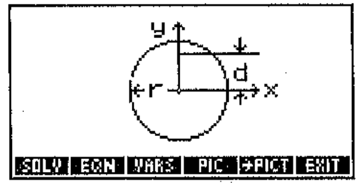
Plane Geometry (11)

Variable Names and Descriptions

β	Central angle of polygon
θ	Vertex angle of polygon
A	Area
b	Base length (Rectangle, Triangle), or Length of semiaxis in x direction (Ellipse)
C	Circumference
d	Distance to rotation axis in y direction
h	Height (Rectangle, Triangle), or Length of semiaxis in y direction (Ellipse)
I, Ix	Moment of inertia about x axis
Id	Moment of inertia in x direction at d
Iy	Moment of inertia about y axis
J	Polar moment of inertia at centroid
L	Side length of polygon
n	Number of sides
P	Perimeter
r	Radius
ri, ro	Inside and outside radii
rs	Distance to side of polygon
rv	Distance to vertex of polygon
v	Horizontal distance to vertex

Reference: 4.

Circle (11, 1)



Equations:

$$A = \pi \cdot r^2 \qquad C = 2 \cdot \pi \cdot r \qquad I = \frac{\pi \cdot r^4}{4}$$

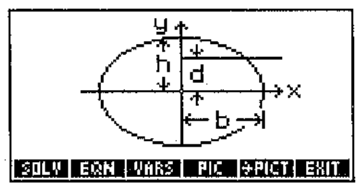
$$J = \frac{\pi \cdot r^4}{2} \qquad Id = I + A \cdot d^2$$

Example:

Given: r=5_cm, d=1.5_cm.

Solution: C=31.4159_cm, A=78.5398_cm^2, I=4908738.5_mm^4, J=9817477.0_mm^4, Id=6675884.4_mm^4.

Ellipse (11, 2)



Equations:

$$A = \pi \cdot b \cdot h \qquad C = 2 \cdot \pi \cdot \sqrt{\frac{b^2 + h^2}{2}} \qquad I = \frac{\pi \cdot b \cdot h^3}{4}$$

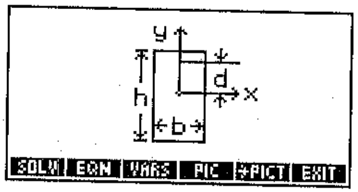
$$J = \frac{\pi \cdot b \cdot h}{4} \cdot \left(b^2 + h^2\right) \qquad Id = I + A \cdot d^2$$

Example:

Given: b=17.85_μm, h=78.9725_μin, d=.00000012_ft.

Solution: A=1.1249E−6_cm^2, C=7.9805E−3_cm, I=1.1315E−10_mm^4, J=9.0733E−9_mm^4, Id=1.1330E−10_mm^4.

Rectangle (11, 3)



Equations:

$$A = b \cdot h \qquad P = 2 \cdot b + 2 \cdot h \qquad I = \frac{b \cdot h^3}{12}$$

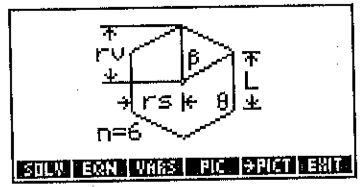
$$J = \frac{b \cdot h}{12} \cdot \left(b^2 + h^2\right) \qquad Id = I + A \cdot d^2$$

Example:

Given: b=4_chain, h=7_rd, d=39.26_in.
Set guesses for I, J, and Id in km^4.

Solution: A=28328108.2691_cm^2, P=23134.3662_cm, I=2.9257E−7_km^4, J=1.8211E−6_km^4, Id=2.9539E−7_km^4.

Regular Polygon (11, 4)



Equations:

$$A = \frac{\frac{1}{4} \cdot n \cdot L^2}{\mathrm{TAN}\left(\frac{180}{n}\right)} \qquad P = n \cdot L \qquad rs = \frac{\frac{L}{2}}{\mathrm{TAN}\left(\frac{180}{n}\right)}$$

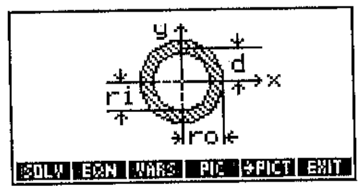
$$rv = \frac{\frac{L}{2}}{\mathrm{SIN}\left(\frac{180}{n}\right)} \qquad \theta = \frac{n-2}{n} \cdot 180 \qquad \beta = \frac{360}{n}$$

Example:

Given: n=8, L=.5_yd.

Solution: A=10092.9501_cm^2, P=365.7600_cm, rs=55.1889_cm, rv=59.7361_cm, θ=135_°, β=45_°

Circular Ring (11, 5)



Equations:

$$A = \pi \cdot \left(ro^2 - ri^2\right) \qquad I = \frac{\pi}{4} \cdot \left(ro^4 - ri^4\right)$$

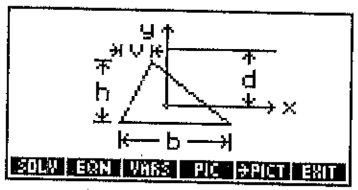
$$J = \frac{\pi}{2} \cdot \left(ro^4 - ri^4\right) \qquad Id = I + A \cdot d^2$$

Example:

Given: ro=4_μ, ri=25.0_kÅ, d=.1_mil.

Solution: A=3.0631E−7_cm^2, I=1.7038E−10_mm^4, J=3.4076E−10_mm^4, Id=3.0648E−10_mm^4.

Triangle (11, 6)



Equations:

$$A = \frac{b \cdot h}{2} \qquad P = b + \sqrt{v^2 + h^2} + \sqrt{(b - v)^2 + h^2}$$

$$Ix = \frac{b \cdot h^3}{36} \qquad Iy = \frac{b \cdot h}{36} \cdot \left(b^2 - b \cdot v + v^2\right)$$

$$J = \frac{b \cdot h}{36} \cdot \left(h^2 + b^2 - b \cdot v + v^2\right) \qquad Id = Ix + A \cdot d^2$$

Example:

Given: h=4.33012781892_in, v=2.5_in, P=15_in, d=2_in.

Solution: b=5.0000_in, Ix=11.2764_in^4, Iy=11.2764_in^4, J=22.5527_in^4, A=10.8253_in^2, Id=54.5776_in^4.

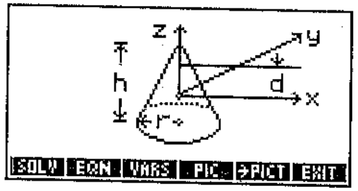
Solid Geometry (12)

Variable Names and Descriptions

A	Total surface area
b	Base length
d	Distance to rotation axis in z direction
h	Height in z direction (Cone, Cylinder), or Height in y direction (Parallelepiped)
I, Ixx	Moment of inertia about x axis
Id	Moment of inertia in x direction at d
Izz	Moment of inertia about z axis
m	Mass
r	Radius
t	Thickness in z direction
V	Volume

Reference: 4.

Cone (12, 1)



Equations:

$$V = \frac{\pi}{3}\cdot r^2\cdot h \qquad A = \pi\cdot r^2 + \pi\cdot r\cdot\sqrt{r^2+h^2}$$

$$Ixx = \frac{3}{20}\cdot m\cdot r^2 + \frac{3}{80}\cdot m\cdot h^2 \qquad Izz = \frac{3}{10}\cdot m\cdot r^2$$

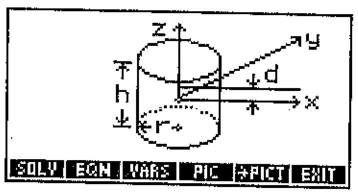
$$Id = Ixx + m\cdot d^2$$

Example:

Given: r=7_cm, h=12.5_cm, m=12.25_kg, d=3.5_cm.

Solution: V=641.4085_cm^3, A=468.9953_cm^2, Ixx=0.0162_kg*m^2, Izz=0.0180_kg*m^2, Id=0.0312_kg*m^2.

Cylinder (12, 2)



Equations:

$$V = \pi\cdot r^2\cdot h \qquad A = 2\cdot\pi\cdot r^2 + 2\cdot\pi\cdot r\cdot h$$

$$Ixx = \frac{1}{4}\cdot m\cdot r^2 + \frac{1}{12}\cdot m\cdot h^2 \qquad Izz = \frac{1}{2}\cdot m\cdot r^2$$

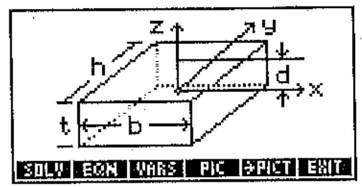
$$Id = Ixx + m\cdot d^2$$

Example:

Given: r=8.5_in, h=65_in, m=12000_lbs, d=2.5_in.

Solution: V=14753.7045_in^3, A=3925.4200_in^2, Ixx=4441750_lb*in^2, Izz=433500_lb*in^2, Id=4516750_lb*in^2.

Parallelepiped (12, 3)



Equations:

$$V = b\cdot h\cdot t \qquad A = 2\cdot\left(b\cdot h + b\cdot t + h\cdot t\right)$$

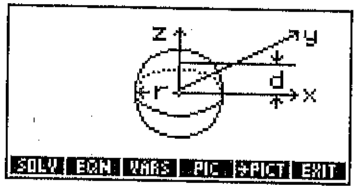
$$I = \frac{1}{12}\cdot m\cdot\left(h^2+t^2\right) \qquad Id = I + m\cdot d^2$$

Example:

Given: b=36_in, h=12_in, t=72_in, m=83_lb, d=7_in.

Solution: V=31104_in^3, A=7776_in^2, I=36852_lb*in^2, Id=40919_lb*in^2.

Sphere (12, 4)



Equations:

$$V = \frac{4}{3}\cdot\pi\cdot r^3 \qquad A = 4\cdot\pi\cdot r^2 \qquad I = \frac{2}{5}\cdot m\cdot r^2 \qquad Id = I + m\cdot d^2$$

Example:

Given: d=14_cm, m=3.75_kg, Id=486.5_lb*in^2.

Solution: r=21.4273_cm, V=41208.7268_cm^3, A=5769.5719_cm^2, I=0.0689_kg*m^2.

Solid State Devices (13)

Variable Names and Descriptions

αF	Forward common-base current gain
αR	Reverse common-base current gain
γ	Body factor
λ	Modulation parameter
μn	Electron mobility
ϕp	Fermi potential
ΔL	Length adjustment (PN Step Junctions), or Channel encroachment (NMOS Transistors)
ΔW	Width adjustment (PN Step Junctions), or Width contraction (NMOS Transistors)
a	Channel thickness
Aj	Effective junction area
BV	Breakdown voltage
Cj	Junction capacitance per unit area
Cox	Silicon dioxide capacitance per unit area
$E1$	Breakdown-voltage field factor
$Emax$	Maximum electric field
$G0$	Channel conductance
gds	Output conductance
gm	Transconductance
I	Diode current
IB	Total base current
IC	Total collector current
$ICEO$	Collector current (collector-to-base open)
ICO	Collector current (emitter-to-base open)
ICS	Collector-to-base saturation current
ID, IDS	Drain current
IE	Total emitter current
IES	Emitter-to-base saturation current

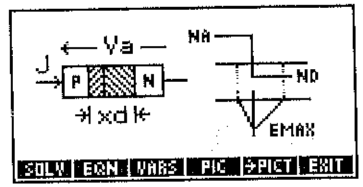
Variable Names and Descriptions (continued)

IS	Transistor saturation current
J	Current density
Js	Saturation current density
L	Drawn mask length (PN Step Junctions), or Drawn gate length (NMOS Transistors), or Channel length (JFETs)
Le	Effective gate length
NA	P-side doping (PN Step Junctions), or Substrate doping (NMOS Transistors)
ND	N-side doping (PN Step Junctions), or N-channel doping (JFETs)
T	Temperature
tox	Gate silicon dioxide thickness
Va	Applied voltage
VBC	Base-to-collector voltage
VBE	Base-to-emitter voltage
Vbi	Built-in voltage
VBS	Substrate voltage
VCEsat	Collector-to-emitter saturation voltage
VDS	Applied drain voltage
VDsat	Saturation voltage
VGS	Applied gate voltage
Vt	Threshold voltage
Vt0	Threshold voltage (at zero substrate voltage)
W	Drawn mask width (PN Step Junctions), or Drawn width (NMOS Transistors), or Channel width (JFETs)
We	Effective width
xd	Depletion-region width
xdmax	Depletion-layer width
xj	Junction depth

References: 5, 8.

PN Step Junctions (13, 1)

These equations for a silicon PN-junction diode use a "two-sided step-junction" model—the doping density changes abruptly at the junction. The equations assume the current density is determined by minority carriers injected across the depletion region and the PN junction is rectangular in its layout. The temperature should be between 77 and 500 K. (See "SIDENS" in chapter 3.)



Equations:

$$\mathrm{Vbi} = \frac{k\ T}{q}\cdot \mathrm{LN}\left(\frac{\mathrm{NA}\cdot\mathrm{ND}}{\mathrm{ni}^2}\right)$$

$$\mathrm{xd} = \sqrt{\frac{2\cdot\epsilon\mathrm{si}\cdot\epsilon 0}{q}\cdot(\mathrm{Vbi}-\mathrm{Va})\cdot\left(\frac{1}{\mathrm{NA}}+\frac{1}{\mathrm{ND}}\right)}$$

$$\mathrm{Cj} = \frac{\epsilon\mathrm{si}\cdot\epsilon 0}{\mathrm{xd}} \qquad \mathrm{Emax} = \frac{2\cdot\left(\mathrm{Vbi}-\mathrm{Va}\right)}{\mathrm{xd}}$$

$$\mathrm{BV} = \frac{\epsilon\mathrm{si}\cdot\epsilon 0\cdot \mathrm{E1}^2}{2\cdot q}\cdot\left(\frac{1}{\mathrm{NA}}+\frac{1}{\mathrm{ND}}\right) \qquad \mathrm{J} = \mathrm{Js}\cdot\left(e^{\frac{q\cdot\mathrm{Va}}{k\cdot T}}-1\right)$$

$$\mathrm{Aj} = \left(\mathrm{W}+2\cdot\Delta\mathrm{W}\right)\left(\mathrm{L}+2\cdot\Delta\mathrm{L}\right) + \pi\cdot\left(\mathrm{W}+\mathrm{L}+2\cdot\Delta\mathrm{W}+2\cdot\Delta\mathrm{L}\right)\cdot\mathrm{xj}+2\cdot\pi\cdot\mathrm{xj}^2$$

$$\mathrm{I} = \mathrm{J}\cdot\mathrm{Aj}$$

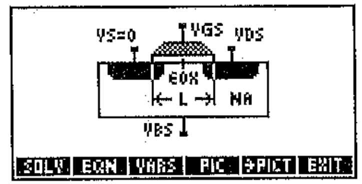
Example:

Given: *ND*=1E22_cm^−3, *NA*=1E15_1/cm^3, *T*=26.85_°C, *Js*=1E−6_μA/cm^2, *Va*=−20_V, *E1*=3.3E5_V/cm, *W*=10_μ, *ΔW*=1_μ, *L*=10_μ, *ΔL*=1_μ, *xj*=2_μ.

Solution: *Vbi*=.9962_V, *xd*=5.2551_μ, *Cj*=2005.0141_pF/cm^2, *Emax*=79908.5240_V/cm, *BV*=358.0825_V, *J*=−1.0E−12_A/cm^2, *Aj*=3.1993E−6_cm^2, *I*=−3.1993E−15_mA.

NMOS Transistors (13, 2)

These equations for a silicon NMOS transistor use a two-port network model. They include linear and nonlinear regions in the device characteristics and are based on a gradual-channel approximation (the electric fields in the direction of current flow are small compared to those perpendicular to the flow). The drain current and transconductance calculations differ depending on whether the transistor is in the linear, saturated, or cutoff region. The equations assume the physical geometry of the device is a rectangle, second-order length-parameter effects are negligible, short-channel, hot-carrier, and velocity-saturation effects are negligible, and subthreshold currents are negligible. (See “SIDENS Function” in chapter 3.)



Equations:

$$\mathrm{We} = \mathrm{W} - 2\cdot\Delta\mathrm{W} \qquad \mathrm{Le} = \mathrm{L} - 2\cdot\Delta\mathrm{L} \qquad \mathrm{Cox} = \frac{\epsilon\mathrm{ox}\cdot\epsilon 0}{\mathrm{tox}}$$

$$\mathrm{IDS} = \mathrm{Cox}\cdot\mu\mathrm{n}\cdot\left(\frac{\mathrm{We}}{\mathrm{Le}}\right)\cdot\left((\mathrm{VGS}-\mathrm{Vt})\cdot\mathrm{VDS} - \frac{\mathrm{VDS}^2}{2}\right)\cdot(1+\lambda\cdot\mathrm{VDS})$$

$$\gamma = \frac{\sqrt{2\cdot\epsilon\mathrm{si}\cdot\epsilon 0\cdot\mathrm{q}\cdot\mathrm{NA}}}{\mathrm{Cox}}$$

$$\mathrm{Vt} = \mathrm{Vt0} + \gamma\cdot\left(\sqrt{2\cdot\mathrm{ABS}(\phi\mathrm{p}) + \mathrm{ABS}(\mathrm{VBS})} - \sqrt{2\cdot\mathrm{ABS}(\phi\mathrm{p})}\right)$$

$$\phi\mathrm{p} = \frac{-\mathrm{k}\cdot\mathrm{T}}{\mathrm{q}}\cdot\mathrm{LN}\left(\frac{\mathrm{NA}}{\mathrm{ni}}\right) \qquad \mathrm{gds} = \mathrm{IDS}\cdot\lambda$$

$$\mathrm{gm} = \sqrt{\mathrm{Cox}\cdot\mu\mathrm{n}\cdot\left(\frac{\mathrm{We}}{\mathrm{Le}}\right)\cdot\left(1+\lambda\cdot\mathrm{VDS}\right)\cdot 2\cdot\mathrm{IDS}}$$

$$\mathrm{VDsat} = \mathrm{VGS} - \mathrm{Vt}$$

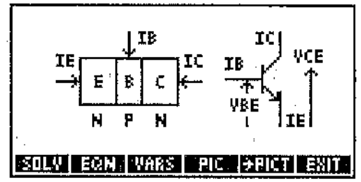
Example:

Given: *tox*=700_Å, *NA*=1E15_1/cm^3, *μn*=600_cm^2/(V*s), *T*=26.85_°C, *Vt0*=.75_V, *VGS*=5_V, *VBS*=0_V, *VDS*=5_V, *W*=25_μ, *ΔW*=1_μ, *L*=4_m, *ΔL*=.75_μ, λ=.05_1/V.

Solution: *We*=23_μ, *Le*=2.5_μ, *Cox*=49330.4750_pF/cm^2, γ=.3725_V^.5, *φp*=−.2898_V, *Vt*=.75_V, *VDsat*=4.25_V, *IDS*=3.0741_mA, *gds*=1.5370E−4_S, *gm*=1.4466_mA/V.

Bipolar Transistors (13, 3)

These equations for an NPN silicon bipolar transistor are based on large-signal models developed by J.J. Ebers and J.L. Moll. The offset-voltage calculation differs depending on whether the transistor is saturated or not. The equations also include the special conditions when the emitter-base or collector-base junction is open, which are convenient for measuring transistor parameters.



Equations:

$$\mathrm{IE} = -\mathrm{IES}\cdot\left(e^{\frac{\mathrm{q}\cdot\mathrm{VBE}}{\mathrm{k}\cdot\mathrm{T}}} - 1\right) + \alpha\mathrm{R}\cdot\mathrm{ICS}\cdot\left(e^{\frac{\mathrm{q}\cdot\mathrm{VBC}}{\mathrm{k}\cdot\mathrm{T}}} - 1\right)$$

$$\mathrm{IC} = -\mathrm{ICS}\cdot\left(e^{\frac{\mathrm{q}\cdot\mathrm{VBC}}{\mathrm{k}\cdot\mathrm{T}}} - 1\right) + \alpha\mathrm{F}\cdot\mathrm{IES}\cdot\left(e^{\frac{\mathrm{q}\cdot\mathrm{VBE}}{\mathrm{k}\cdot\mathrm{T}}} - 1\right)$$

$$\mathrm{IS} = \alpha\mathrm{F}\cdot\mathrm{IES} \qquad \mathrm{IS} = \alpha\mathrm{R}\cdot\mathrm{ICS} \qquad \mathrm{IB}+\mathrm{IE}+\mathrm{IC} = 0$$

$$\mathrm{ICO} = \mathrm{ICS}\cdot\left(1-\alpha\mathrm{F}\cdot\alpha\mathrm{R}\right) \qquad \mathrm{ICEO} = \frac{\mathrm{ICO}}{1-\alpha\mathrm{F}}$$

$$\mathrm{VCEsat} = \frac{\mathrm{k}\cdot\mathrm{T}}{\mathrm{q}}\cdot\mathrm{LN}\left(\frac{1+\frac{\mathrm{IC}}{\mathrm{IB}}\cdot\left(1-\alpha\mathrm{R}\right)}{\alpha\mathrm{R}\cdot\left(1-\frac{\mathrm{IC}}{\mathrm{IB}}\cdot\left(\frac{1-\alpha\mathrm{F}}{\alpha\mathrm{F}}\right)\right)}\right)$$

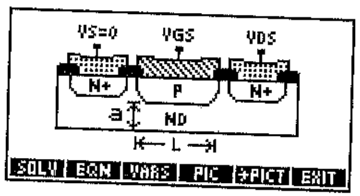
Example:

Given: IES=1E−5_nA, ICS=2E−5_nA, T=26.85_°C, αF=.98, αR=.49, IC=1_mA, VBC=−10_V.

Solution: VBE=.6553_V, IS=0.0000098_nA, ICO=.000010396_nA, $ICEO$=.0005198_nA, IE=−1.0204_mA, IB=.0204_mA, $VCEsat$=0_V.

JFETs (13, 4)

These equations for a silicon N-channel junction field-effect transistor (JFET) are based on the single-sided step-junction approximation, which assumes the gates are heavily doped compared to the channel doping. The drain-current calculation differs depending on whether the gate-junction depletion-layer thickness is less than or greater than the channel thickness. The equations assume the channel is uniformly doped and end effects (such as contact, drain, and source resistances) are negligible. (See "SIDENS" in chapter 3.)



Equations:

$$\mathrm{Vbi} = \frac{\mathrm{k}\cdot\mathrm{T}}{\mathrm{q}}\cdot\mathrm{LN}\left(\frac{\mathrm{ND}}{\mathrm{ni}}\right)$$

$$\mathrm{xdmax} = \sqrt{\frac{2\cdot\epsilon\mathrm{si}\cdot\epsilon0}{\mathrm{q}\cdot\mathrm{ND}}\cdot(\mathrm{Vbi}-\mathrm{VGS}+\mathrm{VDS})}$$

$$\mathrm{G0} = \mathrm{q}\cdot\mathrm{ND}\cdot\mu\mathrm{n}\cdot\left(\frac{\mathrm{a}\cdot\mathrm{W}}{\mathrm{L}}\right)$$

$$\mathrm{ID} = \mathrm{G0}\cdot\left(\mathrm{VDS}-\frac{2}{3}\cdot\sqrt{\frac{2\cdot\epsilon\mathrm{si}\cdot\epsilon0}{\mathrm{q}\cdot\mathrm{ND}\cdot\mathrm{a}^2}}\cdot\left((\mathrm{Vbi}-\mathrm{VGS}+\mathrm{VDS})^{\frac{3}{2}}-(\mathrm{Vbi}-\mathrm{VGS})^{\frac{3}{2}}\right)\right)$$

$$\mathrm{VDsat} = \frac{\mathrm{q}\cdot\mathrm{ND}\cdot\mathrm{a}^2}{2\cdot\epsilon\mathrm{si}\cdot\epsilon0}-(\mathrm{Vbi}-\mathrm{VGS}) \qquad \mathrm{Vt} = \mathrm{Vbi}-\frac{\mathrm{q}\cdot\mathrm{ND}\cdot\mathrm{a}^2}{2\cdot\epsilon\mathrm{si}\cdot\epsilon0}$$

$$\mathrm{gm} = \mathrm{G0}\cdot\left(1-\sqrt{\frac{2\cdot\epsilon\mathrm{si}\cdot\epsilon0}{\mathrm{q}\cdot\mathrm{ND}\cdot\mathrm{a}^2}\cdot(\mathrm{Vbi}-\mathrm{VGS})}\right)$$

Example:

Given: ND=1E16_1/cm^3, W=6_μ, a=1_μ, L=2_μ, μn=1248_cm^2/(V*s), VGS=−4_V, VDS=4_V, T=26.85_°C.

Solution: Vbi=.3493_V, $xdmax$=1.0479_μ, $G0$=5.9986E−4_S, ID=.2268_mA, $VDsat$=3.2537_V, Vt=−7.2537_V, gm=.1462_mA/V.

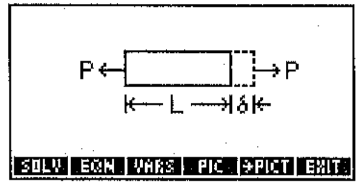
Stress Analysis (14)

Variable Names and Descriptions

δ	Elongation
ϵ	Normal strain
γ	Shear strain
ϕ	Angle of twist
σ	Normal stress
$\sigma 1$	Maximum principal normal stress
$\sigma 2$	Minimum principal normal stress
σavg	Normal stress on plane of maximum shear stress
σx	Normal stress in x direction
$\sigma x1$	Normal stress in rotated-x direction
σy	Normal stress in y direction
$\sigma y1$	Normal stress in rotated-y direction
τ	Shear stress
τmax	Maximum shear stress
$\tau x1y1$	Rotated shear stress
τxy	Shear stress
θ	Rotation angle
$\theta p1$	Angle to plane of maximum principal normal stress
$\theta p2$	Angle to plane of minimum principal normal stress
θs	Angle to plane of maximum shear stress
A	Area
E	Modulus of elasticity
G	Shear modulus of elasticity
J	Polar moment of inertia
L	Length
P	Load
r	Radius
T	Torque

Reference: 2.

Normal Stress (14, 1)



Equations:

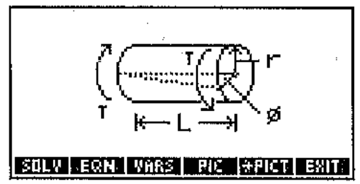
$$\sigma = \mathrm{E} \cdot \epsilon \qquad \epsilon = \frac{\delta}{\mathrm{L}} \qquad \sigma = \frac{\mathrm{P}}{\mathrm{A}}$$

Example:

Given: P=40000_lbf, L=1_ft, A=3.14159265359_in^2, E=10E6_psi.

Solution: δ=0.0153_in, ϵ=0.0013, σ=12732.3954_psi.

Shear Stress (14, 2)



Equations:

$$\tau = \mathrm{G} \cdot \gamma \qquad \gamma = \frac{\mathrm{r} \cdot \phi}{\mathrm{L}} \qquad \tau = \frac{\mathrm{T} \cdot \mathrm{r}}{\mathrm{J}}$$

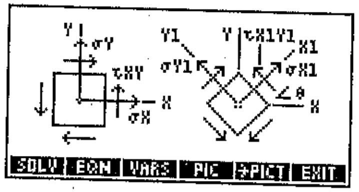
Example:

Given: L=6_ft, r=2_in, J=10.4003897419_in^4, G=12000000_psi, τ=12000_psi.

Solution: T=5200.1949_ft*lbf, ϕ=2.0626_°, γ=5.7296E-2_°

Stress on an Element (14, 3)

Stresses and strains are positive in the directions shown.



Equations:

$$\sigma x1 = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cdot \mathrm{COS}(2 \cdot \theta) + \tau xy \cdot \mathrm{SIN}(2 \cdot \theta)$$

$$\sigma x1 + \sigma y1 = \sigma x + \sigma y$$

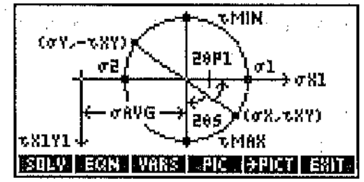
$$\tau x1y1 = -\left(\frac{\sigma x - \sigma y}{2}\right) \cdot \mathrm{SIN}(2 \cdot \theta) + \tau xy \cdot \mathrm{COS}(2 \cdot \theta)$$

Example:

Given: σx=15000_kPa, σy=4755_kPa, τxy=7500_kPa, θ=30_°

Solution: $\sigma x1$=18933.9405_kPa, $\sigma y1$=821.0595_kPa, $\tau x1y1$=−686.2151_kPa.

Mohr's Circle (14, 4)



Equations:

$$\sigma 1 = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}$$

$$\sigma 1 + \sigma 2 = \sigma x + \sigma y$$

$$\mathrm{SIN}(2 \cdot \theta p1) = \frac{\tau xy}{\sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}}$$

$$\theta p2 = \theta p1 + 90 \qquad \tau max = \frac{\sigma 1 - \sigma 2}{2}$$

$$\theta s = \theta p1 - 45 \qquad \sigma avg = \frac{\sigma x + \sigma y}{2}$$

Example:

Given: σx=−5600_psi, σy=−18400_psi, τxy=4800_psi.

Solution: $\sigma 1$=−4000_psi, $\sigma 2$=−20000_psi, $\theta p1$=18.4349_°, $\theta p2$=108.4349_°, τmax=8000_psi, θs=−26.5651_°, σavg=−12000_psi.

Waves (15)

Variable Names and Descriptions

β	Sound level
λ	Wavelength
ω	Angular frequency
ρ	Density of medium
B	Bulk modulus of elasticity
f	Frequency
I	Sound intensity
k	Angular wave number
s	Longitudinal displacement at x and t
sm	Longitudinal amplitude
t	Time
v	Speed of sound in medium (Sound Waves), or Wave speed (Transverse Waves, Longitudinal Waves)
x	Position
y	Transverse displacement at x and t
ym	Transverse amplitude

Reference: 3.

Transverse Waves (15, 1)

Equations:

$$y = \mathrm{ym} \cdot \mathrm{SIN}\left(k \cdot x - \omega \cdot t\right) \qquad v = \lambda \cdot f \qquad k = \frac{2 \cdot \pi}{\lambda} \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: ym=6.37_cm, k=32.11_r/cm, x=.03_cm, ω=7000_r/s, t=1_s.

Solution: f=1114.0846_Hz, λ=.0020_cm, y=2.6655_cm, v=218.0006_cm/s.

Longitudinal Waves (15, 2)

Equations:

$$s = \mathrm{sm} \cdot \mathrm{COS}\left(k \cdot x - \omega \cdot t\right) \qquad v = \lambda \cdot f \qquad k = \frac{2 \cdot \pi}{\lambda} \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: sm=6.37_cm, k=32.11_r/cm, x=.03_cm, ω=7000_r/s, t=1_s.

Solution: s=5.7855_cm, v=2.1800_m/s, λ=.1957_cm, f=1114.0846_Hz.

Sound Waves (15, 3)

Equations:

$$v = \sqrt{\frac{B}{\rho}} \qquad I = \frac{1}{2} \cdot \rho \cdot v \cdot \omega^2 \cdot \mathrm{sm}^2$$

$$\beta = 10 \cdot \mathrm{LOG}\left(\frac{I}{I0}\right) \qquad \omega = 2 \cdot \pi \cdot f$$

Example:

Given: sm=10_cm, ω=6000_r/s, B=12500_kPa, ρ=65_kg/m^3.

Solution: v=438.5290_m/s, I=5130789412.97_W/m^2, β=217.1018_dB, f=954.9297_Hz.

References

1. Dranchuk, P.M., R.A. Purvis, and D.B. Robinson. "Computer Calculations of Natural Gas Compressibility Factors Using the Standing and Katz Correlation," In *Institute of Petroleum Technical Series*, no. IP 74-008. 1974.

2. Gere, James M., and Stephen P. Timoshenko. *Mechanics of Materials*, 2d ed. PWS Engineering, Boston, 1984.

3. Halliday, David, and Robert Resnick. *Fundamentals of Physics*, 3d ed. John Wiley & Sons, 1988.

4. Meriam, J.L., and L.G. Kraige. *Engineering Mechanics*, 2d ed. John Wiley & Sons, 1986.

5. Muller, Richard S., and Theodore I. Kamins. *Device Electronics for Integrated Cicuits*, 2d ed. John Wiley & Sons, 1986.

6. Serghides, T.K. "Estimate Friction Factor Accurately," In *Chemical Engineering*, Mar. 5, 1984.

7. Siegel, Robert, and John Howell. *Thermal Radiation Heat Transfer*, Vol. 1. National Aeronautics and Space Administration, 1968.

8. Sze, S. *Physics of Semiconductors*, 2d ed. John Wiley & Sons, 1981.

9. Welty, Wicks, and Wilson. *Fundamentals of Momentum, Heat and Mass Transfer*. John Wiley & Sons, 1969.

A

Error and Status Messages

In the following tables, messages are first arranged alphabetically by name and then numerically by message number.

Messages Listed Alphabetically

Message	Meaning	# (hex)
Acknowledged	Alarm acknowledged.	619
All Variables Known	No unknowns to slove for.	E405
Autoscaling	Calculator is autoscaling x- and/or y- axis.	610
Awaiting Server Cmd.	Indicates Server mode active.	C0C
Bad Argument Type	One or more stack arguments were incorrect type for operation.	202
Bad Argument Value	Argument value out of operation's range.	203
Bad Guess(es)	Guess(es) supplied to HP Solve application or ROOT lie outside domain of equation.	A01